Significant Contributions by the nominee during his career in India

Sarkar’s research interests are focused on deepening our understanding of bounded linear operators on Hilbert spaces (note that the bounded linear operators on Hilbert spaces are natural generalization/analogue of matrices). More specifically, Sarkar’s research contribution has been in the areas of (module approach to) multivariable operator theory, function theory in both one and several complex variables and the classical one variable operator theory. His research illuminates the relationship between four different mathematical traditions: operator theory, commutative algebra, complex geometry and several complex variables. Here are some highlights of his scientific contributions during his career in India:

(1) **Factorizations in operator theory and function theory:** Sarkar proved a series of results relating factors of kernel functions, multipliers, dilations and invariant subspaces of reproducing kernel Hilbert spaces (see [12], [13], [14] and [15]). He has contributed, by sharpening Arveson’s dilation theorem, to the factorizations of dilations for commuting tuples of operators (see [13]). This idea was developed for contractions on Hilbert spaces (single variable) in the context of comparing the classical Sz.-Nagy and Foias dilations with Agler and Young’s dilations for pairs of commuting operators on Hilbert spaces for which the symmetrized bidisc is a spectral set. This yields, in particular, (i) a complete classification, in terms of a complete set of unitary invariants, of $\Gamma$-contractions (answering a question raised by Jim Agler and Nicholas Young in 2003) [17], and (ii) unique representations of invariant subspaces in one and several variables [13, 21, 22].

(2) **Invariant subspaces:** The existence of invariant subspaces (submodules) of bounded linear operators on Hilbert spaces (Hilbert modules) is a long standing open problem. To this problem, Sarkar obtained an invariant subspace theorem which provides an abstract representation of invariant subspaces of bounded linear operators (both in one and several variables) (see [21] and [22]). His results are more definite in the setting of reproducing kernel Hilbert spaces. Another remarkable result is an abstract characterization of invariant subspaces for $n$-tuples of shifts on vector-valued Hardy space over the unit polydisc in $\mathbb{C}^n$, $n > 1$ (see [3]). As a byproduct, by revealing the complicated structure of commuting tuples of isometries, he obtained a complete classification of a large class of $n$-tuples of commuting isometries. He solved (see [23]) the problem of complete classification of invariant subspaces of $\Gamma$-isometries (which appears in the context of commuting pairs of bounded linear operators on Hilbert spaces for which the symmetrized bidisc is a spectral set), a question raised by Jim Agler and Nicholas Young in 2003. Other notable results includes (i) representations of contractively embedded invariant subspaces in several variables (following the classical result of Luis de Branges, see [6]), and (ii) the fact that the unitarily equivalent submodules are rare (see [13]). From the computational point of view, Sarkar (i) proved that the ranks of co-doubly commuting submodules and two variables Rudin’s submodules are 2 (answering a pair of questions posed by Douglas and Yang. See [9] and [27]), and (ii) obtained an explicit co-rank formula (see [28]) for Rudin quotient modules (this also point out and correct an error in the proof of the main result by Izuchi et. al. (JFA 2011)).

(3) **Function theory and operator theory:** In this context, Sarkar contributed to: (i) A complete classification of Toeplitz and asymptotic Toeplitz operators in several variables which is also an initiation to the notion of asymptotic Toeplitz operators on the unit polydisc [8]. Asymptotic Toeplitz operators yields a complete classification of compact perturbation
of Toeplitz operators in several variables. (iii) A complete classification of elements in the symmetrized polydisc [18]. (iii) In his recent preprint [1], complementing earlier results on dynamics of unilateral weighted shifts, Sarkar obtained a sufficient (but not necessary, with supporting examples) condition for hypercyclicity, mixing and chaos for $M_z^*$, the adjoint of $M_z$, on vector-valued analytic reproducing kernel Hilbert spaces $\mathcal{H}$ in terms of the derivatives of kernel functions on the open unit disc $\mathbb{D}$ in $\mathbb{C}$. Here $M_z$ denotes the multiplication operator by the coordinate function $z$, that is, $(M_z f)(w) = w f(w)$ for all $f \in \mathcal{H}$ and $w \in \mathbb{D}$. This includes the special case of quasi-scalar reproducing kernel Hilbert spaces and hypercyclicity for sums of reproducing kernels in the sense of Aronszajn. His work also yields a complete characterization of hypercyclicity of $M_z^*$ on tridiagonal reproducing kernel Hilbert spaces and some special classes of vector-valued analytic reproducing kernel Hilbert spaces.

(4) **Multivariable dilation theory and von Neumann inequality:** Sarkar’s recent papers presents a taste of dilations, wandering subspaces, inner functions and von Neumann inequality in several variables. This includes (i) a new and relatively explicit proof of Ando’s dilation theorem (see [12] and [16]), (ii) a sharper version of two variables von Neumann inequality [12] (in the sense of distinguished varieties following Agler and McCarthy), (iii) the dichotomy of (the success and failure of the) isometric dilations and (sharper in the sense of algebraic varieties) von Neumann inequalities for commuting $n$-tuples of contractions, $n \geq 3$ [5], and (iv) representations of wandering subspaces of commuting tuples of operators ([13] and [32]).

(5) **Complex geometry and operator theory:** Complex geometry and operator theory was (explicitly) introduced by Cowen and Douglas in the late 70’s. In this line, Sarkar proved an useful result on curvature inequality and explored the similarity and quasi-affinity problems for Hilbert modules in the Cowen-Douglas class in terms of the free modules and the complex geometric objects namely, the hermitian anti-holomorphic vector bundles and curvatures (see [2], [34] and [36]). Sarkar is the initiator of the study of several variables simple Jordan blocks in the setting of unit polydisc ([24] and [30]). Along this line, he obtained definite results on boundary representations (following Arveson), rigidity and essential normality of submodules and quotient modules in several variables [24]. It is very surprising that the essential normality of the cross commutators holds only in two variables [24].

(6) **Operator-valued analytic functions (including polynomials):** Purely contractive and operator-valued analytic functions (known as characteristic functions) are naturally associated with bounded linear operators on Hilbert spaces (see the classical theory of Sz.-Nagy and Foias, de Branges, de Branges and Rovnyak, Nikolski, and Gohberg and Krein). As a continuation of his early work “Contractions with polynomial characteristic functions I. Geometric approach, Transaction of American Math. Society, 2012 (with C. Foias)”, in [11] Sarkar further analyzed the class of contraction with polynomial characteristic functions and proved that such operator-valued analytic functions, or the characteristic functions, admits simple factorizations. The non-trivial factors of such factorizations highlights the role of nilpotent operators in the theory of bounded linear operators on Hilbert spaces. Another remarkable contribution is a several variables analogue of Sz.-Nagy and Foias classical factorization theorem for operator-valued analytic functions [15]. His results along this line will be treated as the first initiative to the theory of factorizations of several variables operator-valued analytic functions on the unit ball of $\mathbb{C}^n$. 
References

[1] M. Aneesh and J. Sarkar, Linear dynamics in reproducing kernel Hilbert spaces, preprint [1st revision has been submitted to Bulletin des Sciences Mathématiques]


